Maximally Using GPS Observation for Water Vapor Tomography

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Abstract—GPS-based water vapor tomography has been proved to be a cost-effective means of obtaining spatial and temporal distribution of atmospheric water vapor. In previous studies, the tomography height is empirically selected without considering the actual characteristics of the local water vapor distribution, and most existing studies only consider the signals passing from the top boundary of the tomography area. Therefore, the observed signals coming out from the side face of the tomography area are excluded as ineffective information, which not only reduces the utilization rate of signals used but also decreases the number of voxels crossed by rays. This becomes the research point of this paper, which studies the possibility of selecting a reasonable tomography boundary and using signals passing from the side face of the tomography area. This paper first tries to determine the tomography height based on the local atmospheric physical property using many years of radiosonde data, and 8 km is selected as the tomography boundary in Hong Kong. The second part focuses on superimposing the signals penetrating from the side face of the tomography area to tomography modeling by introducing a scale factor that is able to determine the water vapor content of each signal with the part that belongs in the tomography area. Finally, a tomography experiment is carried out based on data provided by the Satellite Positioning Reference Station Network (SatRef) in Hong Kong to validate the proposed method. Experimental result demonstrates that the utilization rate of the signal used and the number of voxels crossed by rays are both increased by 30.32% and 12.62%, respectively. The comparison of tomographic integrated water vapor (IWV) derived from different schemes with that from radiosonde and ECMWF data shows that the RMS error of the proposed method (4.1 and 5.1 mm) is smaller than that of the previous method (5.1 and 5.6 mm). In addition, the tomographic water vapor densities derived from different schemes is also compared with those of by radiosonde and ECMWF; the statistical result over the experimental period shows that the proposed method has an average RMS error of 1.23 and 2.12 g/m³, respectively, which is superior to the previous method at 1.60 and 2.43 g/m³, respectively.

Index Terms—European Centre for Medium-Range Weather Forecasts (ECMWF), radiosonde, tomography boundary, utilization rate, water vapor tomography.
geometrical linear estimation of the water vapor content inside the grid for the side traversing rays throughout an empirically exponential negative function. However, there are some points that need to be discussed in these studies like some studies do not give the detailed information about how those signals, which cross out from the model’s side face, are used, and the accuracy and reasonability of empirically exponential negative function or the data selected also remain to be verified. Yao et al. [57] also proposed a method to use the signals crossing out from the side face of the tomography area by introducing the water vapor unit index. However, such behavior needs the support of the radiosonde data, and the established water vapor unit index model cannot be updated for each step of tomography resolution. In addition, not all signals coming out from the side face of the tomography area can be used because not all the water vapor unit index model of different layers can be established.

Generally, most existing studies only used GPS-derived SWV, which passes from the top boundary of the research area. As for the signals penetrating from its side, it is often considered to be useless information and is directly excluded. This behavior not only neglects the contribution of signals passing from the side face of the tomography area to the final tomographic result but also decreases the utilization rate of observed information and the number of voxels crossed by rays. Here, utilization rate refers to the ratio between the signals used as input information for water vapor tomography and all the observed signals. To maximize the utilization rate of existing observations, as well as the number of voxels with SWV signals crossing them, a novel method that considers the signals penetrating from the model’s side face is proposed by introducing the scale factor when building a tomographic observation equation. The proposed method is of the ability to use all signals penetrating from the model’s side face to build the observation equation without using any external information. In addition, the established scale factor model can be updated with the updating of SWV observations for each step of tomography resolution. In addition, a reasonable tomography boundary is determined based on radiosonde data recorded over many years rather than from experience.

This paper is organized as follows. Section II presents the tomographic algorithm used. Details of the determination of an optimized tomography boundary, as well as the method of imposing signals crossing from the side face of the research area to tomography modeling, are described in Section III. In Section IV, the tomographic experiment is carried out and compared with radiosonde data. The discussion and conclusions are presented in Section V.

II. PRINCIPLE OF TOMOGRAPHIC ALGORITHM

Usually, two kinds of tomographic results would be achieved based on different GPS-derived input observations. They are atmospheric wet refractivity and water vapor density, which are based on the slant wet delay (SWD) and SWV, respectively [4], [15], [22], [39], [46], [50]. In this paper, we try to assimilate water vapor information into the NWP system for further study; thus, the SWV observations are selected as tomography input data.

A. Retrieval of SWV

The SWV can be converted using SWD based on the formula as follows [10]:

$$SWV = \frac{10^5}{(k_3/T_m + k'_2)} \cdot R_v \cdot SWD$$  \hspace{1cm} (1)

where $k_3 = 16.52$ K/hPa, $k'_2 = 461.495$ J/K/kg, and $R_v$ is the specific gas constant for water vapor with a value of $3.776 \times 10^2$ K$^2$/hPa. $T_m$ represents the weighted mean temperature of the atmospheric column. Usually, $T_m$ is calculated from the observed surface temperature through an empirical relationship restricted by radiosonde or reanalysis data [9]. In our study, $T_m$ is determined according to an empirical formula $T_m = 272.4 + 0.556 \cdot T_0$ between $T_m$ and surface temperature $T_0$ proposed by Liu et al. [31] with the standard deviation (std) of 1.7 K. SWD is the SWD that may be given as

$$SWD = f_w(\text{ele}) \cdot ZWD + f_w(\text{ele}) \cdot \cot(\text{ele})$$

$$\cdot \left( G_{WE}^W \cdot \sin(\phi) + G_{NS}^W \cdot \cos(\phi) \right) + R$$  \hspace{1cm} (2)

where ele and $\phi$ are the satellite elevation angle and the azimuth angle, respectively; $f_w$ is the wet mapping function; and $G_{WE}$ and $G_{NS}$ are the wet delay gradients in the east–west and north–south directions. Here, $R$ refers to the undifferenced post-fit residual. ZWD is the zenith wet delay, which is extracted from the zenith tropospheric delay (ZTD) by excluding the zenith hydrostatic delay (ZHD) as follows:

$$ZWD = ZTD - ZHD.$$  \hspace{1cm} (3)

An accurate ZHD would be derived using the observed surface pressure based on the empirical model proposed by Saastamoinen [49]:

$$ZHD = \frac{0.002277 \cdot P_s}{1 - 0.00266 \cdot \cos(2\varphi) - 0.00028 \cdot H}$$  \hspace{1cm} (4)

where $P_s$ represents the surface pressure (unit: hPa), and $\varphi$ and $H$ are the station latitude and the geodetic height, respectively.

In this paper, GAMIT/GLOBK (v10.5) [25], [29] is used for processing the GPS data; the interval of ZTD estimated is 30 min, whereas the wet delay gradients in the east–west and north–south directions are estimated once every 2 h, and then the SWV is calculated based on (2)–(4) by combining the meteorological parameters. To reduce the strong correlation of tropospheric parameters in the regional network due to the similar propagation paths of signals existed between ground-based receivers and satellites, three IGS stations (SHAO, LHAZ, and BJFS) with baselines longer than 500 km are introduced [44]. The elevation angle of 10$^\circ$ is determined to avoid the tomographic result being subject to the influence of signal bending [34]. The mapping function aforementioned is the wet Niell mapping function. Since GAMIT/GLOBK only gives double-differenced residuals, the zero difference residual is therefore calculated according to the method proposed by Alber et al. [2], which is based on the assumption that the sum of the single-differenced residuals from two stations in a baseline to all
observed satellites is zero, and the sum of the undifferenced residuals to one satellite for all available stations is also zero [12].

\[ \text{SWV} = 10^{-6} \cdot \int \rho(s) ds \]  

(5)

where \( \rho(s) \) is the water vapor density (unit: g/m\(^3\)), and \( s \) refers to the receiver–satellite trajectory (unit: m). To obtain 3-D water vapor information, the research area is usually discretized into a number of voxels in the horizontal and vertical directions. One assumption is proposed such that the water vapor density in each voxel is a constant during a given period; therefore, the linear equation between SWV and water vapor density can be obtained as follows:

\[ \text{SWV} = \sum_{ijk} (a_{ijk} \cdot x_{ijk}) \]  

(6)

where \( i, j, k \) represent the position of the area of interest in the longitude, latitude, and vertical directions, respectively; \( a_{ijk} \) represents the distance traveled by a satellite signal to a receiver in discretized voxel \( (i, j, k) \), which is calculated based on two intersections between the signal and grid faces; and \( x_{ijk} \) is the water vapor density of voxel \( (i, j, k) \). The matrix form of this observation equation can be rewritten as follows:

\[ y_{m \times 1} = A_{m \times n} \cdot x_{n \times 1} = 0_{m \times 1} \]  

(8)

where \( H \) and \( V \) are the coefficient matrices of horizontal and vertical constraints, respectively. To get the inverse matrix shown in (8), singular value decomposition is used, as described elsewhere [22], [24], [41], [48].

III. MAXIMALLY USING DATA OBSERVED BY GPS

When the geographic location of the tomographic area is determined, on the one hand, the vertical tomography boundary should be determined according to the actual water vapor changes with altitude because the tomography height affects the utilization rate of data observed by ground-based receivers [15]. The higher the tomography boundary, the less the signals penetrate from the top of the tomography area. On the other hand, due to the influence of the selected location of the research area and the specified geometric location of the ground-based receivers and satellite constellation, the signals received come not only from the top of the tomography area but also from its side. However, most existing studies only considered those signals penetrating from the top boundary of the tomography area while neglecting the contribution of signals crossing from its side to the final tomographic result. To maximize use of the data observed by GPS, details of selection of a reasonable tomographic height and how to superimpose the signals, which pass from the model’s side face, to construct the tomographic observation equation are described in this section.

A. Determination of Vertical Optimized Tomography Boundary

Nearly all atmospheric water vapor is concentrated in the troposphere [36]; however, there is no uniform expression describing the relationship between water vapor and altitude. Previously, the tomography height is empirically selected as, for example, 10 or 15 km [22], [40], [53], [55]. However, for different research areas, the distribution of atmospheric water vapor at different altitudes may vary widely. If a high tomography boundary is selected, more unknown water vapor parameters with values close to zero are introduced to the tomography modeling, as well as a reduction in the number of signals penetrating from its top. On the contrary, a low tomography boundary may ignore the influence of water vapor brought above its top, which would lead to a relatively large water vapor field being seen in the tomographic result.

Radiosonde is one of the most accurate means of obtaining the vertical water vapor profile [1], [37]. Therefore, in this section, the idea of determining the optimal tomography height based on the radiosonde is proposed as follows: Many years of radiosonde data in research area are selected to get the vertical water vapor profile and std at different altitudes, and then the
Fig. 1. Average water vapor density and std at different heights.

Fig. 2. Geographic distribution of ground-based receivers in the Hong Kong Satellite Positioning Reference Station Network (SatRef) and the radiosonde station.

tomography height is determined according to the water vapor density value and std for the top layer if less than a given threshold. In our study, thresholds of 0.2 and 0.05 g/m³ are selected for the top layer of the average water vapor density and std in Hong Kong, respectively. Fig. 1 shows the average water vapor density value and std at different altitudes based on radiosonde data from 1974 to 2014. It is shown that the average water vapor density and std are less than the thresholds of 0.2 and 0.05 g/m³ above a height of 8 km, respectively, hence, the height of 8 km is determined as having formed tomography boundary.

The number of signals used and the number of voxels crossed by rays are analyzed on the basis of the 12 stations derived from SatRef (see Fig. 2) for the period of day of year (Doy) 124–150, 2013. The selected tomography area covers latitudes from N21.19° to N21.54° and longitudes from E113.87° to E114.35°; two selected vertical tomography boundaries are set at 8 and 10.4 km, respectively. Voxel division means that the resolutions, in latitude and longitude directions, are 0.06° and 0.05°, respectively, whereas the vertical resolution is in nonuniform vertical layer schemes: a) two layers with a thickness of 500 m, three layers of 700 m, three layers of 900 m, and two layers of 1100 m; and b) two layers with a thickness of 500 m, three layers of 700 m, three layers of 900 m, two layers of 1100 m, and two layers of 1200 m.

Fig. 3 shows the actual 3-D signal distribution received by ground-based receivers under the aforementioned conditions at dates 12:00 UTC Doy 124, 2013. The shades of blue and red indicate the number of times each voxel is crossed by rays. It is shown that some signals which crossed from the top of the tomography area [as shown by green lines in Fig. 3(a)] are penetrated from its side [as shown by yellow lines in Fig. 3(b)] while the tomography height is increased from 8 to 10.4 km. In addition, the average number of signals used and the average number of voxels crossed by rays, of every day in a half-hour period for Doy 124–150, 2013, are analyzed (see Fig. 4). It can be concluded that the average number of signals used and the number
of voxels crossed by rays at the height of 0–8 km are increased as the tomography boundary height decreased from 10.4 to 8 km. The statistical result over the experimental period shows that the average utilization rate of signal used is increased by 18.81%, whereas the average number of voxels crossed by signals at the height of 0–8 km is enhanced by 7.43% from 51.46% to 58.89%. Therefore, a reasonable selecting of tomography height is important when looking to improve the utilization rate of signals and the number of voxels crossed by rays.

B. Method of Superimposing Signals Penetrating From the Side of Tomography Area to Tomography Modeling

The tomography modeling was constructed based on SWV signals crossing the entire research area in previous studies without considering rays penetrating from its side. On the one hand, those SWV signals penetrating from the side face of the tomography area also have application value to water vapor tomography. Directly excluding those signals not only reduces the utilization rate of observed data but also leads to many voxels, especially those located at the edge and the middle-low layers of the tomography area, not being crossed by SWV signals, which further decreases the quality of tomographic result. On the other hand, removing this side face traversed observations dramatically reduces the number of low-elevation observations, which contain important information about the water vapor state measured at the lower level of the tomographic grid because most of the water vapor is located at the low troposphere. Therefore, it is crucial to consider the side model observations. This becomes the focus of this section, which tries to impose the signals passing from the side face of the research area to build the tomographic observation equation.

1) Constructing a Tomography Observation Equation Using SWV Signals Penetrating From the Side: In this section, a method is proposed using the SWV signals which penetrate from the side face of the tomography area to construct an observation equation that can overcome the deficiency of previous studies. The proposed method not only improves the utilization rate of observed GPS data but also increased the number of voxels crossed by signals. The basic concept is as follows: The traditional tomography method is first used to get the water vapor densities; for the SWV signals penetrating the entire tomography area, the total water vapor content in those signals is calculated; then, by narrowing the scale of the tomography area to make those SWV signals penetrating from its side and obtaining the proportional relationship between the water vapor content of an SWV signal that belongs in the narrowed tomography area and the total water vapor content of this SWV signal, the water vapor content of SWV signals crossing from the side face of the original tomography area would be calculated based on the proportional relationship obtained above. Therefore, the calculated water vapor content within the area can be used to establish the observation equation for water vapor tomography.

The specific steps used to establish the tomography observation equation using SWV signals penetrating from the side are described as follows.

1) Obtain the initial water vapor density value of the tomography area only using the signals coming out from its top boundary based on the tomography modeling in (8) as aforementioned.

2) Narrow the range of the tomography area in both latitude and longitude directions to make the SWV signals that penetrate from the top boundary of original tomography area cross from the side face of the narrowed tomography area. As shown by the red line in Fig. 5(a) and (c), it is crossed from the top boundary of the original tomography area (the black box) by narrowing the tomography area in the horizontal direction, which makes the red line penetrate from the side face of the tomography area (the blue box).

3) After the original tomography area is narrowed [as shown by the range of blue and green in Fig. 5(a) and (b), respectively], calculate scale factor, which refers to the ratio between the water vapor content of an SWV signal that belongs within the narrowed tomography area and the total water vapor content of this SWV signal.

The specific process used to calculate the scale factor is as follows. As shown in Fig. 5, (c) is the front view of (a), in which it is shown that the signal OQ (red line) penetrates from the entire tomography area but crosses from the side face of the narrowed tomography area [see the blue range in Fig. 5(a) and (c)]. Therefore, the SWV of signal OQ can be obtained by the integral of water vapor density along its path

\[
\text{SWV}^\text{OQ} = \int_{OQ} \rho_{ijk}^0 ds_{OQ} = \sum_{OQ} \rho_{ijk}^0 \cdot d_{ijk} \quad (9)
\]
where $SWV_{oq}$ is the water vapor content of signal OQ, $\rho_{ijk}^0$ is the initial water vapor density value reconstructed in step 1), whereas $d_{ijk}^o$ is the distance traveled along signal OQ in voxel $(i,j,k)$. Similarly, the value of $SWV_{op}$ and $SWV_{pq}$ in Fig. 5(c) can be also calculated based on (9). Therefore, one scale factor $\alpha_{op}$ of signal OQ would be achieved by using the following formula:

$$\alpha_{op} = \frac{SWV_{op}}{SWV_{oq}}$$ (10)

where $SWV_{oq} = SWV_{op} + SWV_{pq}$.

Then, the tomography area was narrowed again [as shown by the green range in Fig. 5(a) and (d)]; the signal OQ is also crossed from the side face of the selected green area. $SWV_{or}$ would be also calculated based on (9), and another scale factor for signal OQ would be obtained as follows:

$$\alpha_{or} = \frac{SWV_{or}}{SWV_{oq}}$$ (11)

The tomography area is gradually narrowed until this ground-based receiver is out with the narrowed area, and thus, all scale factors of signal OQ were obtained by this way.

4) Steps of 1)–3) are repeated for all received signals from every station and the scale factor model established based on the scale factors calculated in Step 3). It was found that the scale factor has an exponential relationship [see Fig. 6(a)] with the height of intersection of the signal and side of the selected area, as shown by $H_{S1}$ and $H_{S2}$ in Fig. 5(c) and (d) by analyzing the data at two epochs of 00:00 and 12:00 UTC during the tested period. Therefore, an exponential relationship between the scale factor and the height is established as follows:

$$\alpha = a + b \cdot \exp\left(\frac{1}{H_S}\right)$$ (12)

where $a$ and $b$ are coefficients of the scale factor, which can be calculated by least square method, whereas $H_S$ represents the height of intersection of the signal and the side of the tomography area. In our study, $a$ and $b$ are updated during every tomography process.

5) For the $SWV$ signal penetrating from the side of the original tomography area [as shown by the black line in Fig. 5(a)], its scale factor can be calculated according to the height of intersection of the signal and the side face of the original tomography area and the established scale factor model. The water vapor content of the $SWV$ signal would be then obtained to allow the establishment of the observation equation. For example, for signal OI which crosses from the side of the original tomography area in Fig. 5(e), the water vapor content of signal that belongs within the original tomography area $SWV_{oe}$ can be calculated as follows:

$$SWV_{oe} = \alpha_{oi} \cdot SWV_{oi}$$ (13)

where $\alpha_{oi}$ is the scale factor of signal OI, calculated based on the established scale factor model using $H_S$ in Fig. 5(e), whereas $SWV_{oi}$ is the total water vapor content of signal OI.

6) Constructing the tomography observation equation using the $SWV$ signals penetrating from the side face of the original tomography area. After all the parts of $SWV$ signals...
that belong within the tomography area are obtained, therefore, another observation equation could be written as

\[
\begin{pmatrix}
a_{11} & \cdots & a_{1n} \\
\vdots & \ddots & \vdots \\
a_{l1} & \cdots & a_{ln}
\end{pmatrix}
\begin{pmatrix}
x_1 \\
\vdots \\
x_n
\end{pmatrix} =
\begin{pmatrix}
\text{SWV}_{s1} \\
\vdots \\
\text{SWV}_{sl}
\end{pmatrix}
\tag{14}
\]

where \(l\) and \(n\) represent the number of SWV signals that pass from the side face of the tomography area and the number of voxels in the interested area, respectively; \(a\) is the distance crossed by signals that penetrates from the model’s side face; and \(\text{SWV}_{s}\) is the part of water vapor content of SWV signal that only belongs within the tomography area.

Consequently, the final tomography modeling the method proposed above is obtained by combining (8) and the matrix form of (14) as follows:

\[
\begin{pmatrix}
A_{mxn} \\
H_{mxn} \\
V_{mxn} \\
As_{mxn}
\end{pmatrix}
\begin{pmatrix}
x_{n+1}
\end{pmatrix} =
\begin{pmatrix}
y_{m\times1} \\
0_{m\times1} \\
0_{m\times1} \\
y_{s\times1}
\end{pmatrix}
\tag{15}
\]

where \(y\) and \(A\) are the traditional column vector of SWV observations and the design matrix of distance transmitted by the rays that come out from the top boundary of the tomography area, respectively; \(As\) is the another coefficient matrix of distances crossed by signals that penetrate from the side face of the tomography area; and \(ys\) is the column vector of the part of the SWV measurements that belongs within the tomography area.

It is shown in (13) that the accuracy of the calculated part of the water vapor content of SWV signals, which belongs inside the tomography area, is determined by the accuracy of the established scale factor model. Therefore, the established scale factor models are analyzed based on data during the tested days (Doy 124–150, 2013). First, according to the method proposed above, coefficients \(a\) and \(b\) of the daily scale factor model at 30-min intervals are fitted using the calculated scale factor at different altitudes based on the observations (the SWV observations which come out from the top boundary of the tomography area). Second, determining the scale factor at different altitudes based on (12). Finally, the comparison is performed between the scale factor estimated by the scale factor model at different altitudes and the scale factor derived from the observations based on (11). The calculated average daily RMS errors of scale factor for 27 days are analyzed [see Fig. 6(b)]. It can be observed that the largest average RMS value is less than 0.07, and the average RMS value over the experimental period is 0.054 by calculation. Statistical result over the experimental period shows that the average water vapor content of SWV signals which pass from the model’s side face is 117.6 mm, whereas the satellite elevation angle is 10°, thus giving an RMS error in the calculated part of the water vapor content of SWV signals that belong within the tomography area of 6.3 mm.

2) Importance Analysis of SWV Signals Penetrating From the Side of the Tomography Area: As aforementioned in Section III-A, 8 km is selected as the tomography height here. Generally, the value of water vapor density is decreased with altitudes; the vertical tomography grid should be set with an uneven resolution following the water vapor expected behavior, with a thinner spacing on the lower layers (more water vapor density) and larger spacing in the higher layers (less water vapor density). Therefore, following this principle, nonuniform thicknesses of vertical voxel layer scheme are determined as follows: two layers with a thickness of 500 m, three layers of 700 m, three layers of 900 m, and two layers of 1100 m.

The following analyzes the importance of SWV signals penetrating from the side of the tomography area. Fig. 7 shows the observed 3-D signal distribution while the elevation angle is 10° at date 12:00 UTC Doy 124, 2013, where (a) only considers the SWV signals (as shown by green lines) penetrating from the top of the research area whereas (b) includes those SWV signals crossing from both the top and side (as shown by green and yellow lines) of the research area. The shades of blue and red represent the number of times a voxel is crossed by SWV signals. It is shown in Fig. 7(a) that many voxels, especially those located at the edge and lower-middle layers, are not crossed by any rays while only considering the signals crossing from the top boundary of the tomography area, and only 57.14% of voxels are crossed by rays. However, if the SWV signals penetrating from its side face are also included, the percentage of voxels crossed by signals is increased by 16.43% to 73.57%. The voxels crossed by the SWV signals that pass from the side are mainly concentrated on the edge and the lower-middle layers of the tomography area [see red shading in Fig. 7(b)], which just makes up for the aforementioned deficiency in previous studies. In addition, atmospheric water vapor is mainly focused in the lower-middle layers of the tomography area, and
Fig. 7. Observed 3-D distribution of GPS signals where (a) only considers the SWV signals penetrating from the top of the research area whereas (b) includes the SWV signals crossing from both the top and side at date 12:00 UTC Doy 124, 2013. Green lines and yellow lines represent signals penetrating from the top and side, respectively, whereas the shades of blue and red represent the number of times a certain voxel crossed by signals.

Fig. 8. Statistical result of the average number of signals used and the number of voxels crossed by rays for schemes 1 and 2 during the period of Doy 124–150, 2013. Having sufficient SWV signals cross those areas is a prerequisite to achieve an accurate tomographic result. Therefore, the SWV signals penetrating from the side are essential when trying to improve the quality of water vapor tomography.

To compare further the number of SWV signals used and the number of voxels crossed by rays, two schemes are designed based on the data and research area aforementioned in Hong Kong. Scheme 1 only considers signals passing the entire research area, whereas Scheme 2 includes signals penetrating from both its top and side boundaries. Fig. 8 shows the average number of signals used and the average number of voxels crossed by rays every day, at 30-min intervals, throughout the experimental period. It is shown that the number of signals used and the number of voxels crossed by rays, derived from Scheme 2, are both larger than those with Scheme 1. Statistical result over the experimental period shows that, when the elevation angle is 10° and considering the SWV signals penetrating from the side face of the tomography area, the average utilization rate of signals is increased by 30.32%, whereas the average number of voxels crossed by signals is enhanced by 12.62% from 58.89% to 71.51%.

IV. COMPARISON WITH WATER VAPOR DENSITY DERIVED FROM RADIOSONDE AND ECMWF

Here, a reasonable tomography height is determined based on many years of radiosonde data, and 8 km is selected for the tomography area in Hong Kong. The SWV signals that pass from the side face of the tomography area are superimposed to construct the observation equation by introducing the scale factor, which then gives the final tomography modeling. However, for any tomography modeling, the accuracy of the tomographic result is key to evaluate the quality of tomography modeling. Therefore, two schemes are used to build the tomography modeling and evaluate the quality of the tomographic result. The two schemes are as follows.

Scheme 1: Only consider the SWV signals penetrating the entire tomography area to build the observation equation and final tomography modeling like (8).

Scheme 2: Consider both SWV signals crossing from the top and side of the tomography area to construct the observation equation and final tomography modeling like (15).

A. IWV Comparison

Here, the accurate vertical water vapor profiles derived from radiosonde data are selected as a reference to evaluate the accuracy of water vapor density derived from other methods [13], [32], [37]. Data of 12 stations derived from SatRef for the period of Doy 124–150, 2013 are selected to carry out the tomography experiment. A direct comparison of IWV was first performed with voxel information obtained from two tomographic schemes and radiosonde data for the location of the radiosonde station at dates 00:00 and 12:00 UTC epochs throughout the experimental period. Fig. 9 shows a comparison...
of IWV results, from which it can be concluded that the IWV time series of Scheme 2 gives better agreement than that from the radiosonde with respect to that of Scheme 1. The statistical results over the experimental period show that the RMS error, maximum difference, and minimum difference of IWV sets between Scheme 2 and radiosonde are 4.1, 7.6, and −10.4 mm, respectively, whereas those of IWV sets between Scheme 1 and radiosonde are 5.1, 8.7, and −10.6 mm, respectively.

In addition, the evaluations of the tomographic IWV results derived from different schemes are also compared using data from the European Centre for Medium-Range Weather Forecasts (ECMWF). ECMWF can provide global reanalysis data four times per day at four dates: UTC 00:00, 06:00, 12:00, and 18:00, which include meteorological elements such as temperature and relative humidity for different layers, and the highest horizontal resolution level is 0.125° × 0.125°. In our study, there are evenly distributed 12 grid points in the tomography area. The IWV for the location of the radiosonde station is constructed using ECMWF grid point data by inverse distance weighted method for UTC 00:00 and 12:00 epochs of the tested days. The IWV series of tomographic results and ECMWF (see Fig. 10) over the experimental days show that Scheme 2 has good agreement with radiosonde and ECMWF data compared with that of Scheme 1 in both epochs. We can find that, for the radiosonde comparison, the RMS values from Scheme 1, on the two dates, are 1.41 and 2.65 g/m³, respectively, whereas Scheme 2 has RMS error of 1.08 and 1.42 g/m³, respectively. As for the ECMWF comparison, the RMS values from Scheme 1, on the two dates, are 2.68 and 2.87 g/m³, respectively, whereas Scheme 2 has RMS error of 2.21 and 2.07 g/m³, respectively.

In addition, the average RMS error of each day at 00:00 and 12:00 UTC epochs is calculated for each of the 27 test days. Fig. 12 shows the average RMS values of different schemes. It is clear that the RMS error of Scheme 2 is smaller than that of Scheme 1 both compared to Radiosonde and ECMWF for the selected days, which shows that the tomography modeling proposed is superior to the previous tomography modeling. The main reason is that more observed signals are used and more voxels are crossed by rays; therefore, more reliable information can be used for tomography.

One point should be pointed out that IWV is merely the integral of water vapor density in vertical direction, which cannot reflect the spatial distribution of 3-D water vapor density. Therefore, the tomography modeling from Scheme 2 may not be superior to that of Scheme 1 because the IWV value is unchanged if two vertical layers are exchanged arbitrarily. In order to prove further the superiority of the proposed method, the vertical water vapor profiles derived from different schemes at two dates, 00:00 UTC Doy 124, 2013 and 12:00 UTC, Doy 137 are selected for comparison with that from radiosonde and ECMWF data (see Fig. 11). Those two epochs are determined as they correspond to the minimum and maximum values of IWV during the experimental period. It is shown in Fig. 11 that the water vapor profile of Scheme 2 is more consistent with radiosonde and ECMWF data compared with that of Scheme 1 in both epochs. We can find that, for the radiosonde comparison, the RMS values from Scheme 1, on the two dates, are 1.41 and 2.65 g/m³, respectively, whereas Scheme 2 has RMS error of 1.08 and 1.42 g/m³, respectively. As for the ECMWF comparison, the RMS values from Scheme 1, on the two dates, are 2.68 and 2.87 g/m³, respectively, whereas Scheme 2 has RMS error of 2.21 and 2.07 g/m³, respectively.

In addition, the average RMS error of each day at 00:00 and 12:00 UTC epochs is calculated is for each of the 27 test days. Fig. 12 shows the average RMS values of different schemes. It is clear that the RMS error of Scheme 2 is smaller than that of Scheme 1 both compared to Radiosonde and ECMWF for the selected days, which shows that the tomography modeling proposed is superior to the previous tomography modeling. The main reason is that more observed signals are used and more voxels are crossed by rays; therefore, more reliable information can be used for tomography. Table I also lists the statistical results between two tomography schemes, Radiosonde, and ECMWF over the tested period. It is shown that, for the radiosonde comparison, the RMS error and Bias of Scheme 2 are 1.23 and −0.07 g/m³, respectively, whereas those of Scheme 1 are 1.60 and −0.16 g/m³, respectively. For the comparison with ECMWF, the RMS error and Bias of Scheme 2 are 2.12 and −1.51 g/m³, respectively, whereas those of Scheme 1 are 2.43 and −1.60 g/m³, respectively. The comparison of ECMWF and radiosonde also implies that the water vapor tomographic result has a better quality (RMS 1.60 g/m³ of Scheme 1 and RMS 1.23 g/m³ of Scheme 2) than the ECMWF data (RMS 2.10 g/m³). Therefore, for local area, it is feasible.
to enhance the accuracy of ECMWF by assimilating the high-quality water vapor tomographic result into the ECMWF.

To investigate further the correlation of vertical water vapor density profile with respect to altitude, relative error was considered according to the following formula [15]:

\[ re = \frac{|x_T - x_{RS/ECMWF}|}{x_{RS/ECMWF}} \]  

(16)

where \( re \) is the relative error, \( x_T \) represents the water vapor density derived from different schemes, and \( x_{RS/ECMWF} \) represents the water vapor density from the radiosonde or ECMWF. RMS and relative errors of each voxel for different altitudes are calculated, and a total of 54 sets of data are selected for the experimental period at each voxel altitude because only two sets of radiosonde data are available each day during the 27 test days.

Fig. 13 shows the comparison of RMS and relative errors with height between different schemes, radiosonde, and ECMWF data. It is a clear evidence that the RMS and relative error of Scheme 2 at most altitudes are smaller than those of Scheme 1, which is enough to suggest that the tomography modeling proposed by imposing the signals passing from the side of the research area is better than the previous tomography, which only uses signals penetrating the whole research area. The maximal values of RMS error between different schemes and radiosonde data are seen in the lowest layers (3.06 g/m³ for Scheme 1 and 2.38 g/m³ for Scheme 2), whereas the relatively large values of relative error occur in both top and bottom. This is because of the large difference between the tomographic and radiosonde data in the lower layers; therefore, a large RMS and relative error are generated therein. For the upper layers, the water vapor density is very low, and even a small difference between the radiosonde and tomographic result can also lead to a large relative error.

In addition to the comparison using radiosonde data, the water vapor profile derived from different schemes are also evaluated using ECMWF data. The results are obtained from the comparison between different schemes and ECMWF (see Fig. 14) that the maximal RMS error both occurred at the 3–4 km (3.54 g/m³ for Scheme 1 and 3.26 g/m³ for Scheme 2), while the large value of relative error happened at the upper layers.

V. DISCUSSION AND CONCLUSION

In this paper, a method of maximally using signals that pass through the tomography area has been proposed for regional ground-based water vapor tomography. In the vertical direction, the tomography boundary is selected based on the
actual atmospheric physical property using 40-year radiosonde data obtained in the local area, which can provide accurate water vapor profiles at different altitudes; therefore, the selected tomography boundary height corresponds to the actual water vapor distribution while guaranteeing more signals penetrating from the top of the tomography area. As for the signals passing from its side, a scale factor is introduced, which is used to determine the proportion of the signal that belongs within the tomography area. Therefore, the signals penetrating from its side are also considered when constructing the observation equation for water vapor tomography. In addition, constraints of horizontal smoothing information and vertical a priori condition are also imposed to any subsequent tomography modeling.

The proposed method of maximally using the data derived from GPS has been validated in tomographic experiments based on the GPS observations and meteorological data derived from SatRef in Hong Kong. Comparisons of the number of signals used and the number of voxels crossed by rays have shown that the utilization rate of signal used is improved by 30.32%, whereas the number of voxels crossed by rays is enhanced by 12.62% while considering the signals penetrating from the side of the tomography area. In addition, the experimental result shows that the RMS error of the proposed method (4.1 and 5.1 mm, respectively) is better than that found when using the previous method (5.1 and 5.6 mm, respectively) according to a comparison with IWV derived from radiosonde and ECMWF data. The average RMS error and bias over the experimental period are also calculated: A comparison of radiosonde, ECMWF, and tomography shows that the RMS errors of the proposed method (1.23 and 2.12 g/m³, respectively) are superior to those of the previous method (1.60 and 2.43 g/m³, respectively). The water vapor density at different altitudes also shows the superiority of the proposed method, and it is found that the RMS error generally decreases with height, whereas the relative error is greater at top and bottom layers.

Although the proposed method increases the number of voxels with crossing signals, many voxels are still not crossed by any signal. By using rays derived from four operational GNSS (GPS, GLONAA, GALILEO, and BDS) in the near future, more voxels are expected to be crossed. Some external sources are also expected to impose into the tomographic system such as radiometers and COSMIC data. Accurate water vapor profiles derived from tomography have many practical applications in precipitation forecasting, in which, by introducing water vapor density values into the existing assimilation and forecasting systems, it is promising that the forecasting ability would be enhanced, especially for short-term and nowcasting precipitation if the real-time water vapor density information is adopted. Therefore, the real-time 4-D water vapor tomography will be considered as our further research work.

ACKNOWLEDGMENT

The authors would like to thank IGAR for providing access to the web-based IGAR data; the Survey and Mapping Office, Lands Department for providing the GPS and meteorological data; and the International GNSS Service (IGS) for providing precise GPS satellite orbit data and IGS stations.

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