A new ionospheric tomography model combining pixel-based and function-based models

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Abstract

Considering the limitations of current single pixel-based and function-based computerized ionospheric tomography (CIT) models, this paper proposes a new tomography model – COMBI, which combines these two models. COMBI model is able to reconstruct the three dimensional distribution of electron density with fewer parameters, and easy to compute, as well as very convenient to use. Through experiments with simulated data and measured data, it is verified that the new COMBI model not only can better describe refine structure of ionospheric electron density, but also is superior to these two pixel-based and function-based CIT models in application.

Keywords: Ionospheric tomography model; Pixel-based model; Function-based models; Total electron content; Ionospheric electron density

1. Introduction

As a part of the Earth’s upper atmosphere, Ionosphere is an important part of the Sun-Earth environment, and a part of the Earth’s upper atmosphere. Ionization composition in ionosphere significantly affects the transmission of electromagnetic waves in the ionosphere, which causes the reflection of electromagnetic waves and the energy loss, thereby affects the activity of modern radio communications and human space.

In order to deeply study the three-dimensional structure of the ionosphere, Austen et al. (1986, 1988) first proposed the thought of Computerized Ionospheric Tomography (CIT). He obtained the spatial distribution image of ionospheric electron density using the scan of the ionospheric region came from polar-orbiting satellite. The ionospheric tomography reconstruction image can not only reflect horizontal change, but also reflect the vertical structure of ionosphere, which overcomes the limitations of single-layer ionospheric model. Many scholars conduct a plenty of studies on radio tomography (RT) method, the results of these experiments are included in the reviews (Kunitsyn and Tereshchenko, 1991; Leitinger, 1999; Pryse, 2003; Bust and Mitchell, 2008) and books (Kunitsyn and Tereshchenko, 1991, 2003; Kunitsyn et al., 2007). Wang et al. (2007) conducted experiments of ionospheric tomography using a new HF radio source. They found that reconstructed images using the HF method gave comparable results with the VHF/UHF method.

Since the Global Positioning System (GPS) established in the late 1970s, ionospheric delay in radio transmission process has caught the widespread concern, which also gave birth to the GPS ionospheric detection technology; and then the study of the ionosphere has been developed by leaps and bounds. A large amount of researchers have successively studied theoretical models and methods of GPS-based Ionospheric Tomography. Now, ionospheric models are broadly divided into two categories: one is the function-based ionospheric model (Gao and Liu, 2002; Hansen, 1998; Howe, 1997; Liu, 2004; Geng, 2011); the
other is the pixel-based ionospheric model (Hernandez-Pajares et al., 1999; Kunitake et al., 1995; Ma and Maruyama, 2003; Rius et al., 1997a; An, 2011; Wen, 2007; Zou, 2004; Zou and Xu, 2003a,b). Jin et al. (2011) presented the current status and new developments of remote sensing using GNSS signals as well as its future directions and applications. The authors concluded that the denser ground GNSS system could monitor more detailed ground surface characteristics and processes and evolutions of the atmospheric and ionospheric profiles at global and regional scales.

In the study of function-based ionospheric tomography model, as early as 1992, Fremouw proposed to express ionospheric vertical model with empirical orthogonal function (EOF), and horizontal model with spherical harmonic function (Fremouw et al., 1992). Then Hansen (1998) first definitely gave function-based ionospheric tomography model, and obtained the ionospheric electron density distribution using the observations of WAAS system. Howe et al. (1998) extended the reconstruction height range to the entire ionosphere, and obtained time-varying three-dimensional ionospheric structure with the help of Kalman filter, using simulated GPS data. Ruffini et al. (1998) proposed correlation function model and achieved a global ionospheric reconstruction with it. Gao and Liu (2002) put forward a real-time function-based model, which improved the computational efficiency by combining the ionosphere TEC smoothing model and the function-based tomography model. Brunini et al. (2004) put forward the tensor product model of spherical harmonics and Chapman profile function. Nesterov and Kunitsyn (2011) applied a method based on the choice of the smoothest solution by minimizing of a certain Sobolev’s norm for the sought-for function to overcome the non-uniqueness of the solution of the problem with incomplete data is suggested.

In the study of pixel-based ionospheric tomography, Rius et al. (1997b) first reconstructed spatial–temporal distribution of global ionospheric electron density, using GPS information offered by IGS to discretize ionosphere into small grids in earth-fixed coordinate system. Hernandez-Pajares et al. (1998) successfully obtained global ionospheric electron density distribution with high resolution in clam and disturbed state, respectively; and got ionospheric electron density distribution during geomagnetic storms. Bust et al. (2000) compared the results obtained by pixel-based tomography model using GPS data with simulated results from IRI (International Reference Ionosphere) model and observations from ionosondes, and they attested that GPS-based ionospheric electron density profiles are closer to the profiles obtained from ionosondes. The IRI model is an empirical model based on a wide range of ground and space data, it describes monthly averages of ionospheric densities and temperatures in the altitude range 50–1500 km in the non-auroral ionosphere (Bilitza and Reinisch, 2008). Jin and Park (2007) reconstructed the ionospheric electron density profile of southern Korea in 2003, with the data of Korean GPS Network (KGN, Korean GPS Network); then verified the reliability of the result through a comparison with the profile obtained from ionosonde, they also made a comparison with the result from IRI-2001 model.

When using the pixel-based model to reconstruct ionospheric electron density, it is necessary to discretize ionosphere into a number of small grids, and assume that electron density in each grid is constant, and that TEC in the transmission way is equal to the sum of the products of electron density and corresponding ray-intercept in every grid. Its disadvantage is the number of discretized grids is too large, so describing a three dimensional distribution of one region needs a large amount of parameters, which is very inconvenient to use.

Function-based model (Gao and Liu, 2002; Hansen, 1998; Howe, 1997; Liu, 2004; Geng, 2011), however, utilizes a set of functions to express ionospheric electron density distribution. It is good at describing the distribution of a large region with a very small amount of model parameters; but owing to the limitation of ionospheric tomography itself, it would be very difficult to solve equations if we directly fit the observations.

This paper proposes a new tomography model which combines pixel-based model and function-based model. The new model absorbs advantages of the above two models, as well as avoids disadvantages of them. Hence, it is simple to solve equations and convenient to use. In addition, reliability of the new model is verified with simulated data and real observations.

2. Construction of the new tomography model–COMBI

The ionospheric TEC is the line integral electron density on the signal propagation path. It is expressed as

\[
\text{TEC} = \int_{l} Ne(\mathbf{r}, t) ds
\]  

(1)

where \( Ne \) is the electron density along the signal propagation path \( l \). GNSS-based ionospheric CIT uses a series of TECs along \( l \) to inverse the temporal and spatial distributions of ionospheric electron density. The pixel-based model (Hernandez-Pajares et al., 1999; Kunitake et al., 1995; Ma and Maruyama, 2003; Rius et al., 1997a; An, 2011; Wen, 2007; Zou, 2004; Zou and Xu, 2003a,b) must discretize the reconstruction region. Assuming the electron density in each grid is constant during the reconstruction hours, the discretized equation is shown in (1):

\[
\text{TEC}_i = \sum_{j=1}^{J} A_{ij} x_j + e_i
\]  

(2)

where \( \text{TEC}_i \) is the total electron content of the \( i \)th ray, and in the next experiments we wrote it as STEC (slant TEC), \( A_{ij} \) is the intercept of the \( i \)th ray path traversing the \( j \)th grid, \( x_j \) is the electron density of the \( j \)th grid, and \( J \) is the total number of grids. Assuming the number of rays simultaneously observed is \( m \), Eq. (1) can be generally written in a simple matrix notation as:
where \( y_{m\times 1} \) and \( A_{m\times J} \) is known, \( x_{J \times 1} \) is unknown, and needs to be solved. Ionospheric tomography is a typical inversion problem, however, Eq. (2) is ill-posed, cannot be solved directly. To solve this problem, a series of researches have been developed by scholars in the world. The arithmetic of ionospheric tomography can amend the initial electron density by continuously iterating; finally attain the result close to actuality. But due to the limitation of the inadequate GPS observations, many grids are not passed through by rays in the reconstruction hours, so both vertical and horizontal constraints are necessary to be added according to the relationship of electron densities between adjacent grids. The defect of pixel-based model is that too many discretized grids results in too many data required to describe the three dimensional distribution of electron density. For example, if a region has a span of 20° in latitude and longitude, a range from 100 to 1000 km in altitude, and the interval in latitude and longitude is 2°, in altitude is 50 km; then the region would be divided into \( 11 \times 11 \times 19 = 2299 \) grids. Even though the result can be obtained, it is inconvenient to use.

Function-based model expresses electron density with spherical harmonics (or spherical cap harmonic) in horizontal, and with empirical orthogonal function (EOF) in altitude, shown as Eq. (3). The electron density at one point can be got as soon as inputting its latitude, longitude and altitude. The advantage of this model is the capability of using a small amount of parameters to describe a wide range of spatial and temporal distribution of electron density. For example, only 48 parameters are required when both spherical harmonics and empirical orthogonal function are taken to third-order. However, it is difficult to compute these parameters by directly fitting observations because of the limitation of ionospheric tomography itself.

**3. Validation of COMBI model with simulated data**

The section will validate the feasibility of COMBI model using simulated STEC data, the process to produce simulated STEC data is as follows:

(1) A region is chosen, the longitude ranges from 0° to 20°E, latitude ranges from 40° to 60°N, and altitude ranges from 100 to 1000 km with the step of 2°. The discretized interval in longitude and latitude is 2°.

(2) To make the simulated experiment is closer to actual case, we choose real observation stations and satellites. 56 IGS tracking stations in Western Europe are chosen; then the coordinates of satellites in the inversion hours, and the intercepts of every ray path of the satellite-receiver pair traversing the corresponding grids are computed. The intercepts compose the matrix A.

In order to take the advantage of both CIT models introduced above, a new tomography model combining pixel-based and function-based models is proposed in this paper, which is called COMBI model. The process of constructing it is: firstly, one should accurately obtain the electron densities of all grids which are passed through by rays based on pixel-based model, using classic MART algorithm, which has been detailedly described in references (Wen, 2007; Chen, 2012). In this step, it is unnecessary to add constraint for the needlessness of electron densities of all grids. Secondly, the accurate electron densities \( (Ne) \) of these grids should be fitted with spherical harmonics (or spherical cap harmonic) and empirical orthogonal function. And then the model coefficients can be solved using the least squares method, as Eq. (4):

**Equation (4)**

\[
Ne(\phi, \theta, h) = \sum_{k=1}^{K} \sum_{n=0}^{N} \sum_{m=0}^{n} P_{nm}(\sin \phi) \\
\times \left[ a_{nk}^m \cos(m\theta) + b_{nk}^m \sin(m\theta) \right] Z_k(h) \\
= \sum_{k=1}^{K} \sum_{n=0}^{N} \sum_{m=0}^{n} \left[ a_{nk}^m \cos(m\theta) \tilde{P}_{nm}(\sin \phi) Z_k(h) \right. \\
+ b_{nk}^m \sin(m\theta) \tilde{P}_{nm}(\sin \phi) Z_k(h) \left. \right] \\
= a_{nk}^m \sum_{k=1}^{K} \sum_{n=0}^{N} \cos(m\theta) \tilde{P}_{nm}(\sin \phi) Z_k(h) \\
+ b_{nk}^m \sum_{k=1}^{K} \sum_{n=0}^{N} \sin(m\theta) \tilde{P}_{nm}(\sin \phi) Z_k(h)
\]
The ionospheric electron density $N_e(x_{\text{sim}})$ at the central spot of each grid at UT14:00, December 16th, 2010, is computed, using IRI2007 model (Bilitza and Reinisch, 2008) which can be downloaded on the site of http://iri.gsfc.nasa.gov. Then multiply them by coefficient matrix A, so the STEC($y_{\text{sim}}$) in every propagation path of the satellite-receiver pair is obtained:

$$y_{\text{sim}} = A \cdot x_{\text{sim}}$$

Considering the observation noises and discretization errors in actual situation, a small amount of random error $e_{\text{rand}}$ is added in the current paper:

$$y_{\text{noise}} = y_{\text{sim}} + e_{\text{rand}}$$

In order to verify the reconstruction result of COMB model, COMBI model and SIRT algorithm are used to reconstruct the electron densities, respectively; and then the differences between the results obtained by the two methods and IRI2007 model are compared and analyzed. Fig. 1 shows the comparison result, in which the left one shows the difference between the ionospheric electron density obtained from SIRT and IRI2007, the right one shows the difference between the ionospheric electron density obtained from COMBI and IRI2007. From Fig. 1, we can see that electron density distribution reconstructed by COMBI is consistent with that by SIRT algorithm, and closer to the result using IRI2007; the electron density variation with latitude and altitude is well presented. The biggest difference is at 250 km, which is relevant to the electron density at that altitude. In addition, the differences at the boundary are bigger, while in the interior are smaller, it is relevant to the fewer rays at the boundary.

Table 1 lists the statistical result of the reconstruction differences of two methods. From Table 1, we can see the accuracies of these two methods are almost equivalent. COMB model, however, well describes the three-dimensional distribution of ionospheric electron density with fewer parameters. Additionally, the differences of reconstruction results between two methods and IRI2007 model are relevant to the size of added random error, relaxation factor, and iterative threshold.

For further verify the result of COMBI model in altitude profile, Fig. 2 shows the differences of the results reconstructed by two methods in 300 km altitude profile, the left one shows the difference between SIRT and IRI2007, the right one shows the difference between COMBI and IRI2007. From Fig. 2, we can see that results differences of both methods are slightly different but generally consistent. The differences in the interior are smaller, while at the boundary are bigger, in the northeast region are the biggest. The reason is that observation stations distribution results in fewer rays traversing the boundary area.

Table 2 lists the statistical result of the two methods at different altitude profiles. From Table 2, we can see that the maximum, minimum, and standard deviation of the reconstruction differences of the two methods are very close to each other.

### 4. Validation of COMBI model with measured data

To further validate the COMBI model, it is necessary to verify the reliability of the COMBI model using real observations. In this experiment, the epoch is UT10:00, December 16, 2010, and the region is the same as before-mentioned region in the simulation experiment (40–60°N, 0–20°E, 100–1000 km). The observations in this paper come from the western European IGS continuous tracking station network, and the time interval of GPS observations is 30 s. We select 56 IGS stations, use observations from 51 stations of them to reconstruct ionospheric electron density, observations from the remaining five stations–LAMA, HUEG, POTS, TLSE and WARE to do exterior check; besides, electron density profiles obtained from two ionosondes in the region are used to do independent check, they

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Table 1

<table>
<thead>
<tr>
<th></th>
<th>SIRT algorithm</th>
<th>COMBI model</th>
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<tbody>
<tr>
<td>4°E</td>
<td>5.50</td>
<td>5.43</td>
</tr>
<tr>
<td>10°E</td>
<td>5.33</td>
<td>5.43</td>
</tr>
<tr>
<td>16°E</td>
<td>5.50</td>
<td>5.43</td>
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Table 2

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<tr>
<th></th>
<th>SIRT algorithm</th>
<th>COMBI model</th>
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<tbody>
<tr>
<td>Maximum difference</td>
<td>5.30</td>
<td>5.43</td>
</tr>
<tr>
<td>Minimum difference</td>
<td>-3.17</td>
<td>-3.63</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>0.83</td>
<td>0.91</td>
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</table>
are DB049 (Dourbes, 4.6°E, 50.1°N) and PQ052 (Pruhonice, 14.6°E, 50.0°N). The station distribution is shown in Fig. 3, in which, ▲ represents IGS stations used to reconstruct electron density, ● represents stations used to external check, * represents ionosondes used to independent check.

The interval in latitude and longitude is 2°, in altitude is 50 km. Firstly, we compute the initial electron densities in every grid using IRI2007 model. Then, the TEC obtained with measured data in the signal propagation path is used to iterate and amend the initial electron density. Finally, we get accurate electron density in the region.

Figs. 4 and 5, respectively give the ionospheric electron density distribution at 10°E and 300 km reconstructed with SIRT algorithm and COMBI model, the left one shows the result got by SIRT, and the right one shows result got by COMBI, both of their units are 10^{11}\text{e}/m^3. As can be seen from the figures, the reconstruction results got by two methods are close to each other, without obvious difference, both of them are able to reflect three dimensional distribution of electron density in the inversion region. From the comparison figure in the altitude profile (Fig. 5), we can see that these two results are very close in the interior, while slightly different at the boundary.

Tables 3 and 4, respectively give statistical results in longitude and altitude profiles obtained by the two methods. They show the difference of the results obtained by the two models is small, however, the COMBI model solely use a small amount of parameters to express ionospheric electron density. This verifies the reliability of COMBI model.

This paper calculates STEC on signal propagation paths using the five stations–LAMA, HUEG, POTS, TLSE and WARE, which are not involved in modeling. Further, the STEC obtained from the five stations are compared with STEC calculated with the electron density coming from above tomography reconstruction. The comparison result verifies the reliability of the reconstruction result. Fig. 6 gives the comparison of STEC obtained by reconstruction electron density and real observations. From Fig. 6, we can see that STEC obtained from SIRT and COMBI are very close to that obtained from measured data, the absolute difference is less than 2TECu. Hence, the result of exterior...
check demonstrates the two inversion methods in this paper are reliable.

In addition, this paper uses the NmF2 obtained from ionosondes to do independent check. Fig. 7 and Fig. 8 show comparisons of electron density profiles reconstructed from different methods at where the ionosondes are, and comparisons of NmF2s obtained by these methods with ionosondes. The figures show that the COMBI model result at PQ052 station is closer to NmF2 from the ionosonde. At DB049 station, however, SIRT result is closer to NmF2 from the ionosonde. Therefore, the result of independent check also demonstrates COMBI model is reliable, and its accuracy is similar to SIRT’s. Also can be seen from the figures, comparing with initial values, there are obvious changes near the F2 layer for both SIRT algorithm and COMBI model, while slighter changes below the F2 layer. It is resulted from fewer rays at the bottom of the inversion region, which is relevant with ground-data used in this paper. The inversion quality of the region below the F layer can be improved by adding ionosondes and obscuration data.

Table 3
Statistical result at different longitude profiles using SIRT algorithm and COMBI model (unit:10¹¹ el/m³).

<table>
<thead>
<tr>
<th></th>
<th>SIRT</th>
<th>COMBI</th>
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<tr>
<td></td>
<td>Maximum</td>
<td>Minimum</td>
</tr>
<tr>
<td></td>
<td>(10¹¹ el/m³)</td>
<td>(10⁹ el/m³)</td>
</tr>
<tr>
<td>4°E</td>
<td>5.05</td>
<td>6.18</td>
</tr>
<tr>
<td>10°E</td>
<td>4.88</td>
<td>6.47</td>
</tr>
<tr>
<td>16°E</td>
<td>4.62</td>
<td>6.65</td>
</tr>
</tbody>
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Table 4
Statistical result at different altitude profiles using SIRT algorithm and COMBI model (unit:10¹¹ el/m³).

<table>
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<tr>
<th></th>
<th>SIRT</th>
<th>COMBI</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>Maximum</td>
<td>Minimum</td>
</tr>
<tr>
<td></td>
<td>(10¹¹ el/m³)</td>
<td>(10⁹ el/m³)</td>
</tr>
<tr>
<td>200 km</td>
<td>4.36</td>
<td>1.51</td>
</tr>
<tr>
<td>300 km</td>
<td>4.00</td>
<td>2.41</td>
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<tr>
<td>400 km</td>
<td>1.78</td>
<td>1.05</td>
</tr>
<tr>
<td>500 km</td>
<td>0.91</td>
<td>0.40</td>
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In this paper, a new ionospheric tomography model combining pixel-based and function-based models is proposed. The new model is constructed with two steps, and no additional constraint is needed in the process of computation. It expresses 3-D distribution of electron density with fewer model parameters, very convenient to use, and easy to realize computation as well.

In order to verify the reliability of COMBI model, this paper performs experiments using simulated and measured data. In the experiment with simulated data, we compare reconstruction results from COMBI model and SIRT algorithm with the result from IRI2007 model. And the comparison demonstrates that in different longitude and altitude profiles, ionospheric electron density distribution obtained by COMBI model is in general consistent with which obtained by SIRT algorithm, and is closer to which obtained by IRI2007 model.

Additionally, the COMBI model is constructed using observations of 51 western European IGS continuous tracking stations, and checks of reconstruction results are done using observations of other five stations and two ionosondes. It is verified that the accuracy of our new COMBI model is equal to SIRT method while with a concise expression and convenient application, it is able to reflect the 3-D spatial distribution of electron density in reconstruction region. The exterior and independent checks further demonstrate the reliability of COMBI model.

The COMBI model solved the problems of too many parameters for function-based model. Users can get electron density with the spatial solution they needs only with the model parameters. Additionally, for ionospheric anomalies study, this model can be used in the studies of ionospheric responses for magnetic storms, solar activities, and pre-earthquake ionospheric research. It can extract details of ionospheric anomalies caused by these special phenomena, and so offer more information to understand ionospheric change in unusual circumstances.

Ionospheric 3-D tomographic model is an effective access to reflect the 3-D even 4-D spatial and temporal characteristics of ionosphere, but the inversion of electron density is a significant ill-posed problem, more reconstruction methods to solve this problem are still needed to be studied. Besides, combining LEO satellites observations like COSMIC data will be favorable for refined CIT modeling in areas with insufficient GPS stations. Currently,
almost all methods for CIT modeling do not give a notice to assessing approximation errors when using various estimation methods, some related study should be conducted in the future.

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